ELSEVIER

Available online at www.sciencedirect.com

ScienceDirect

Discrete Applied Mathematics 155 (2007) 1745–1751

DISCRETE APPLIED MATHEMATICS

www.elsevier.com/locate/dam

# A sufficient condition for $P_k$-path graphs being $r$-connected ☆

C. Balbuena[a], P. García-Vázquez[b]

[a]Departament de Matemàtica Aplicada III, Universitat Politècnica de Catalunya, Campus Nord, Edifici C2,C/Jordi Girona 1 i 3, E-08034 Barcelona, Spain

[b]Departamento de Matemática Aplicada I, Universidad de Sevilla, Avda Reina Mercedes 2, E-41012 Sevilla, Spain

Received 4 June 2003; received in revised form 13 April 2005; accepted 5 October 2006

Available online 14 April 2007

**Abstract**

Given an integer $k \geqslant 1$ and any graph $G$, the path graph $P_k(G)$ has for vertices the paths of length $k$ in $G$, and two vertices are joined by an edge if and only if the intersection of the corresponding paths forms a path of length $k-1$ in $G$, and their union forms either a cycle or a path of length $k+1$.

Path graphs were investigated by Broersma and Hoede [Path graphs, J. Graph Theory 13 (1989), 427–444] as a natural generalization of line graphs. In fact, $P_1(G)$ is the line graph of $G$. For $k=1,2$ results on connectivity of $P_k(G)$ have been given for several authors. In this work, we present a sufficient condition to guarantee that $P_k(G)$ is connected for $k \geqslant 2$ if the girth of $G$ is at least $(k+3)/2$ and its minimum degree is at least 4. Furthermore, we determine a lower bound of the vertex-connectivity of $P_k(G)$ if the girth is at least $k+1$ and the minimum degree is at least $r+1$ where $r \geqslant 2$ is an integer.



*Keywords:* Connectivity; $r$-connected; Path graphs

## 1. Introduction

Throughout this paper only undirected simple graphs without loops or multiple edges are considered. Unless stated otherwise, we follow the book by Chartrand and Lesniak [6] for terminology and definitions.

A graph $G$ is said to be *connected* if any two vertices can be joined by a path. A graph $G$ is $r$-connected ($r \geqslant 2$) if either $G$ is a complete graph $K_{r+1}$ or else it has at least $r+2$ vertices and no set of $r-1$ vertices separates it.

The aim of this paper is to study the connectivity of $P_k$-path graphs. Following the notation that Knor and Niepel used in [9], given a positive integer $k$ and a graph $G$, the vertex set of the $P_k$-path graph $P_k(G)$ is the set of all paths of length $k$ of $G$, two vertices of $P_k(G)$ are joined by an edge if and only if the intersection of the corresponding paths forms a path of length $k-1$ in $G$, and their union forms either a cycle or a path of length $k+1$. This means that the vertices are adjacent if and only if one can be obtained from the other by "shifting" the corresponding paths in $G$. It is worth mentioning that path graphs are very related to sequence graphs defined by Fiol et al. [8]. Instead of considering the paths of $G$ of length $k$ as vertices, these authors consider the walks of $G$ (not necessarily different vertices) of length $k$, the adjacency being the same.

☆ Research supported by the Ministry of Education and Science, Spain, and the European Regional Development Fund (ERDF) under project MTM2005-08990-C02-02.

*E-mail addresses:* m.camino.balbuena@upc.edu (C. Balbuena), pgvazquez@us.es (P. García-Vázquez).

0166-218X/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.dam.2007.04.003

Path graphs were introduced by Broersma and Hoede [5] as a natural generalization of line graphs, because for $k=1$ path graphs are just line graphs. Since then, most of the work carried out focused in case $k=2$. Thus, Broersma and Hoede [5] characterized the graphs that are $P_2$-path graphs; a problem with their characterization was resolved by Li and Lin [14]. The determination problem for $P_2$-path graphs is solved in [1,16–18], and distance properties of path graphs are studied in [4,11]. Results on the edge-connectivity of line graphs are given by Chartrand and Stewart [7], later by Zamfirescu [20], and recently by Meng [19]. Recent results on vertex-connectivity of iterated line graphs are provided by Knor and Niepel [12]. The vertex-connectivity of $P_2$-path graphs has been studied by Knor et al. [13] and by Li [15]. From a result showed in [11], it is not difficult to see that if $G$ is a connected graph with at most one vertex of degree one, then $P_2(G)$ is also connected. In [2] the edge-connectivity of $P_2$-graphs is studied giving lower bounds on the edge-connectivity which are expressed in terms of the edge-connectivity of $G$. These latter bounds are generalized in [3] for $k \geqslant 2$. Recently, Knor and Niepel [10] have studied the connectivity of $P_3$-path graphs, and furthermore the following sufficient condition to guarantee connected $P_k$-path graphs for $k \geqslant 2$ is easily derived.

**Theorem A** (*Knor and Niepel [10]*). *Let $k \geqslant 2$ be an integer. Let $G$ be a connected graph of minimum degree $\delta(G) \geqslant 2$ and girth $g(G) \geqslant k+1$. Then $P_k(G)$ is connected.*

In this work, we show that the condition on the girth of Theorem A can be relaxed if the minimum degree is at least 4. Furthermore, we determine a lower bound of the vertex-connectivity of $P_k(G)$ if the girth is at least $k+1$ and the minimum degree is at least $r+1$ where $r \geqslant 2$ is an integer. More precisely we prove the following two theorems.

**Theorem 1.1.** *Let $k \geqslant 2$ be an integer. Let $G$ be a connected graph of minimum degree $\delta(G) \geqslant 4$ and girth $g(G) \geqslant \lceil (k+3)/2 \rceil$. Then $P_k(G)$ is connected.*

**Theorem 1.2.** *Let $k, r \geqslant 2$ be integers. Let $G$ be an $r$-connected graph of minimum degree $\delta(G) \geqslant r+1$ and girth $g(G) \geqslant k+1$. Then $P_k(G)$ is $r$-connected.*

Notice that Theorem 1.2 can be seen as a generalization of Theorem A, because for $r=1$ Theorem 1.2 is just Theorem A.

## 2. Proofs

Let us consider a positive integer $k \geqslant 2$. Let us denote by $U=(u_0u_1\cdots u_k)$ a vertex in $P_k(G)$, and by $U: u_0, u_1, \ldots, u_k$ the corresponding path of length $k$ in $G$. We would like to emphasize that $u_i \neq u_j$ for every pair of vertices included in $U$, and that $U=(u_0u_1\cdots u_k)=(u_ku_{k-1}\cdots u_0)$.

**Lemma 1.** *Let $k \geqslant 2$ be an integer. Let $G$ be a connected graph of minimum degree $\delta(G) \geqslant 4$ and girth $g(G) \geqslant \lceil (k+3)/2 \rceil$. Suppose that $U=(u_0u_1\cdots u_k)$ and $V=(v_0v_1\cdots v_k)$ are two vertices in $P_k(G)$ such that $u_k=v_k$. Then there exists a path from $U$ to $V$ in $P_k(G)$.*

**Proof.** Let $i$ be the smaller integer in $\{1, 2, \ldots, k\}$ such that $u_l=v_l$ for $l=i, \ldots, k$. See Fig. 1.

Notice that it could exist some $s, l \in \{0, \ldots, i-1\}$ in such a way that $\{u_0, \ldots, u_s\} \cap \{v_0, \ldots, v_l\} \neq \emptyset$, see Fig. 2.

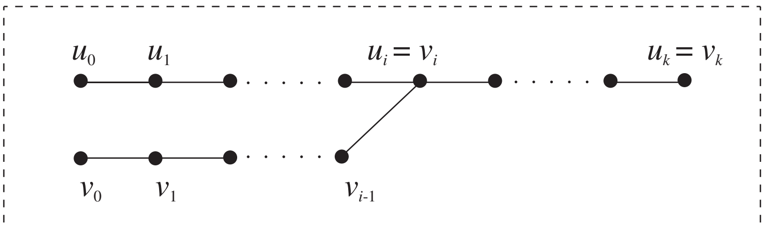


Fig. 1. Two paths in $G$ defining two vertices $U$ and $V$ in $P_k(G)$ with $u_k = v_k$.

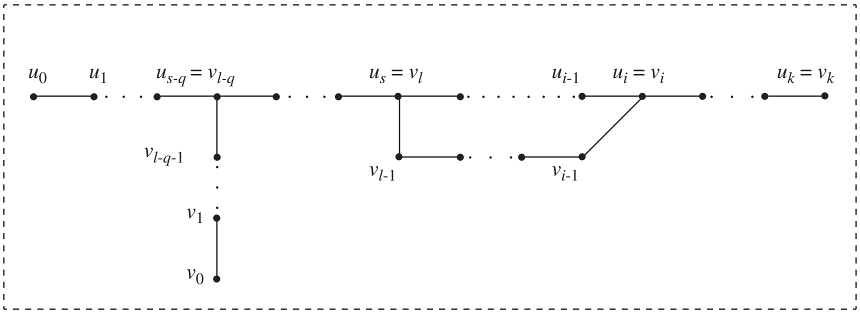


Fig. 2. Two paths in $G$ defining two vertices $U$ and $V$ in $P_k(G)$ with $u_k = v_k$.

First of all, let us see that the graph $G$ contains a path $u_k = t_0, t_1, \ldots, t_i$ such that $t_j \notin \{u_j, u_{j+1}, \ldots, u_k\} \cup \{v_j, v_{j+1}, \ldots, v_k\}$ for all $j = 1, 2, \ldots, i$.

If $k = 2, 3$ the lemma is clear because $\delta(G) \geqslant 4$. Thus assume $k \geqslant 4$. We reason by contradiction assuming that there exists some $j \in \{1, 2, \ldots, i\}$ such that each neighbor $z$ of $t_{j-1}$ satisfies $z \in \{u_j, u_{j+1}, \ldots, u_k\} \cup \{v_j, v_{j+1}, \ldots, v_k\}$. Since $g(G) \geqslant \lceil (k+3)/2 \rceil$ it follows that $N_G(t_{j-1}) \subseteq \{t_{j-2}\} \cup \{u_j, \ldots, u_{j+\lfloor (k-3)/2 \rfloor}\} \cup \{v_j, \ldots, v_{j+\lfloor (k-3)/2 \rfloor}\}$. Since $\delta(G) \geqslant 4$, we may suppose that there are at least two vertices in $\{u_j, \ldots, u_{j+\lfloor (k-3)/2 \rfloor}\}$ adjacent to $t_{j-1}$. This means that $G$ contains a cycle of length at most $\lfloor (k+1)/2 \rfloor$ against the assumption $g(G) \geqslant \lceil (k+3)/2 \rceil$. Therefore, there exists a vertex $t_j \in N_G(t_{j-1})$ such that $t_j \notin \{u_j, u_{j+1}, \ldots, u_k\} \cup \{v_j, v_{j+1}, \ldots, v_k\}$.

As a consequence of the above fact we can find in $P_k(G)$ the following path:

$$\begin{aligned} Z : U &= (u_0 \cdots u_{k-1} t_0), (u_1 \cdots u_{k-1} t_0 t_1), \ldots, \\ &(u_i \cdots u_{k-1} t_0 t_1 \cdots t_i) = (v_i \cdots v_{k-1} t_0 t_1 \cdots t_i), (v_{i-1} \cdots v_{k-1} t_0 t_1 \cdots t_{i-1}), \ldots, \\ &(v_0 \cdots v_{k-1} t_0) = (v_0 \cdots v_{k-1} v_k) = V. \end{aligned}$$

Since the path $Z$ joins vertex $U$ with vertex $V$ in $P_k(G)$ the lemma follows. □

**Proof of Theorem 1.1.** Given two vertices $A = (a_0 a_1 \cdots a_k)$ and $B = (b_0 b_1 \cdots b_k)$ of $P_k(G)$ we will show that $A$ and $B$ can be joined by a path in $P_k(G)$. We distinguish two cases:

*Case* 1: Suppose that $\{a_0, \ldots, a_k\} \cap \{b_0, \ldots, b_k\} \neq \emptyset$.

We may assume that $a_s = b_i$ for some $s, i \in \{0, \ldots, k\}$ such that if $s < k$ then $\{b_0, \ldots, b_k\} \cap \{a_{s+1}, \ldots, a_k\} = \emptyset$. Let $r$, $0 \leqslant r \leqslant s$, be the maximum integer such that:

$$a_{s-j} = b_{i+j} \quad \text{for each } j = 0, \ldots, r,$$

see Fig. 3. That is, we have

$$\{b_{i+r+1}, \ldots, b_k\} \cap \{a_{s-r}, \ldots, a_k\} = \emptyset$$

and

$$\{b_0, \ldots, b_i\} \cap \{a_0, \ldots, a_s\} = \emptyset. \tag{1}$$

Notice that it might be $\{b_{i+r+1}, \ldots, b_k\} \cap \{a_0, \ldots, a_{s-r-1}\} \neq \emptyset$, see Fig. 4.

First suppose $i + s \leqslant k$. Then the walk

$$w_0, w_1, \ldots, w_n = b_k, \ldots, b_{i+r+1}, a_{s-r}, \ldots, a_s, \ldots, a_k$$

is a path because of (1), of length $n = 2k - (i+s) \geqslant k$. Applying Lemma 1 to vertices $A = (a_0 a_1 \cdots a_k)$ and $V_1 = (w_{n-k} \cdots w_n) = (b_{i+s} b_{i+s-1} \cdots b_{i+r+1} a_{s-r} \cdots a_k)$ we can consider in $P_k(G)$ a path $Z_1$ joining $A$ with $V_1$. Moreover, since $n \geqslant k$ we can find in $P_k(G)$ the path

$$Z_2 : V_1 = (w_{n-k} \cdots w_n), \ldots, (w_0 \cdots w_k) = (b_k \cdots b_{i+r+1} a_{s-r} \cdots a_{s+i}) = V_2.$$

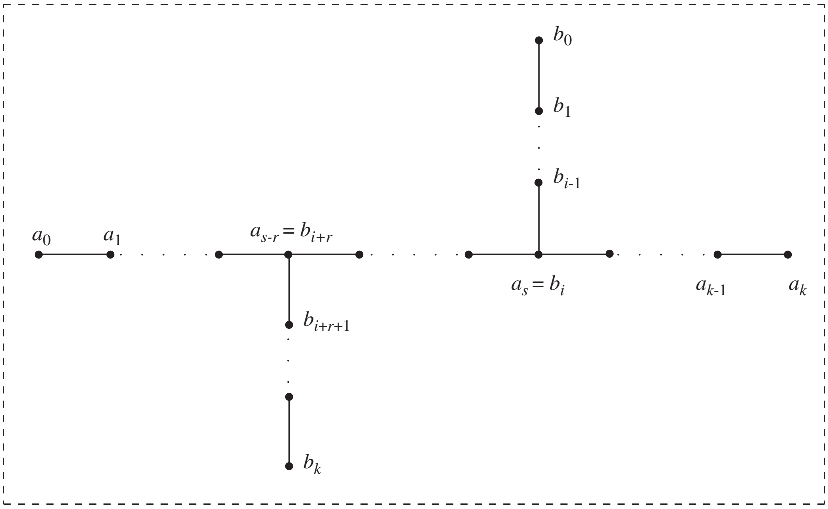


Fig. 3. Detail of paths of $G$ corresponding to vertices $A$ and $B$ of $P_k(G)$.

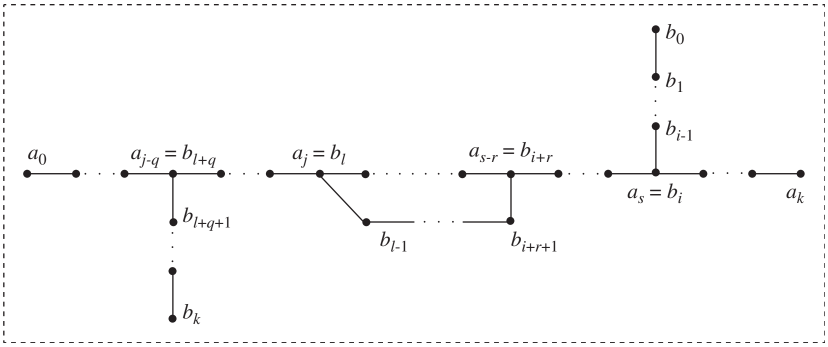


Fig. 4. Detail of paths of $G$ corresponding to vertices $A$ and $B$ of $P_k(G)$.

Notice that if $n = k$ then $V_1 = V_2$ and $Z_2$ is a path of length zero. Finally, by applying Lemma 1 to two vertices $V_2 = (w_k \cdots w_0) = (a_{s+i} \cdots a_{s-r} b_{i+r+1} \cdots b_k)$ and $B = (b_0 b_1 \cdots b_k)$ we obtain that there exists other path $Z_3$ joining these two vertices $V_2$ and $B$. Therefore, the walk $Z_1 \cup Z_2 \cup Z_3$ connects the vertices $A$ and $B$ in $P_k(G)$, hence the theorem holds.

Second, suppose $i + s > k$. Then the walk

$$w_0, w_1, \ldots, w_n = a_0, \ldots, a_{s-1}, b_i, \ldots, b_0$$

is a path because of (1) of length $n = i + s > k$. Applying Lemma 1 to vertices $B = (b_k b_1 \cdots b_0)$ and $V_1 = (w_{n-k} \cdots w_n) = (a_{s-(k-i)} \cdots a_{s-1} b_i \cdots b_0)$ we can consider in $P_k(G)$ a path $Z_1$ joining $B$ with $V_1$. Moreover, since $n > k$ we can find in $P_k(G)$ the path

$$Z_2 : V_1 = (w_{n-k} \cdots w_n), \ldots, (w_0 \cdots w_k) = (a_0 \cdots a_{s-1} b_i \cdots b_{i-(k-s)}) = V_2.$$

Finally, by applying Lemma 1 to two vertices $V_2 = (w_k \cdots w_0) = (b_{i-(k-s)} \cdots b_i a_{s-1} \cdots a_0)$ and $A = (a_k a_{k-1} \cdots a_0)$ we get that there exists other path $Z_3$ joining these two vertices $V_2$ and $A$. Therefore, the walk $Z_1 \cup Z_2 \cup Z_3$ connects the vertices $B$ and $A$ in $P_k(G)$, and the theorem holds.

*Case* 2: $\{a_0, \ldots, a_k\} \cap \{b_0, \ldots, b_k\} = \emptyset$.

Since $G$ is a connected graph, there exists in $G$ a path

$$Z : a_l = z_0, z_1, \ldots, z_h = b_k,$$

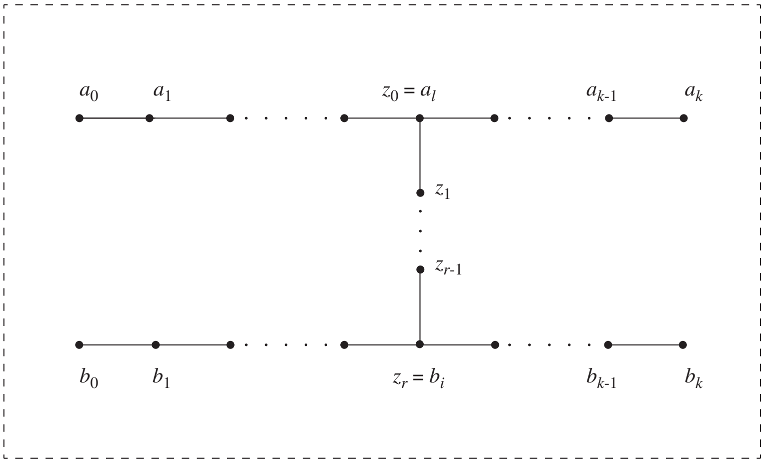


Fig. 5. Detail of disjoint paths of $G$ corresponding to vertices $A$ and $B$ in $P_k(G)$.

joining $a_l$ $(l \in \{0, \ldots, k\})$ with $b_k$ in such a way $\{z_1, \ldots, z_h\} \cap \{a_0, \ldots, a_k\} = \emptyset$. Let $z_r = b_i$ with $r \geqslant 1$ and $i \in \{0, \ldots, k\}$ be such that $\{z_0, \ldots, z_{r-1}\} \cap \{b_0, \ldots, b_k\} = \emptyset$, see Fig. 5. Notice that we may assume $l + i - r \leqslant k$, because otherwise it is enough to interchange $a_\beta$ with $a_{k-\beta}$ or $b_\beta$ with $b_{k-\beta}$ or both. Then the walk

$$w_0, \ldots, w_n = a_k, \ldots, a_l, z_1, \ldots, z_r, b_{i+1}, \ldots, b_k$$

is in fact a path of $G$ of length $n = 2k + r - l - i \geqslant k$. By applying Lemma 1 to vertices $A = (a_0 a_1 \cdots a_k)$ and $W_1 = (w_k \cdots w_0) = (w_k \cdots a_l \cdots a_k)$ there exists in $P_k(G)$ a path $Z_1$ joining $A$ with $W_1$. Moreover, since $n \geqslant k$ we can find in $P_k(G)$ the path

$$Z_2 : W_1 = (w_k \cdots a_l \cdots a_k), (w_{k+1} \cdots a_l \cdots a_{k-1}), \ldots, (w_n \cdots w_{n-k}) = W_2.$$

Observe that if $n = k$ then $W_1 = W_2$ and $Z_2$ is of length 0. Finally, by applying Lemma 1 to vertices $W_2 = (w_{n-k} \cdots w_n) = (w_{n-k} \cdots b_i \cdots b_k)$ and $B = (b_0 b_1 \cdots b_k)$ there exists other path $Z_3$ joining these two vertices $W_2$ and $B$. Consequently, the walk $Z_1 \cup Z_2 \cup Z_3$ connects the vertices $A$ and $B$ in $P_k(G)$, and the theorem holds. □

**Proof of Theorem 1.2.** In order to prove the result we apply Menger's Theorem, i.e., given two vertices of $P_k(G)$, say $A = (a_0 \cdots a_k)$ and $B = (b_0 \cdots b_k)$, we will show that there exist $r$ internally vertex-disjoint paths in $P_k(G)$ joining $A$ with $B$.

Since $b_k \neq b_0$ we may assume that $a_k \neq b_k$. As $G$ is $r$-connected, by applying Menger's Theorem, there exist $r$ internally vertex-disjoint paths in $G$ connecting $a_k$ with $b_k$. Let us denote these paths by

$$Z_i : a_k = z_0^i, z_1^i, \ldots, z_{h_i}^i = b_k \quad \text{for } i = 1, \ldots, r.$$

First, let us find $r - 2$ internally vertex-disjoint paths in $P_k(G)$ joining $A$ with $B$. Since the paths $Z_i$ are internally vertex-disjoint we have

$$z_1^i \neq a_{k-1} \quad \text{and} \quad z_{h_i-1}^i \neq b_{k-1} \quad \text{for } i = 1, \ldots, r-2.$$

Moreover, due to $g(G) \geqslant k + 1$ we have

$$z_j^i \notin \{a_j, a_{j+1}, \ldots, a_k\} \quad \text{for } 1 \leqslant j \leqslant \min\{h_i, k\}, \quad i = 1, \ldots, r-2. \tag{2}$$

$$z_{h_i-j}^i \notin \{b_j, b_{j+1}, \ldots, b_k\} \quad \text{for } 1 \leqslant j \leqslant \min\{h_i, k\}, \quad i = 1, \ldots, r-2. \tag{3}$$

Let us consider the walks

$$\mathscr{P}_i : a_0, a_1, \ldots, a_k, z_1^i, \ldots, z_{h_i-1}^i, b_k, b_{k-1}, \ldots, b_0, \quad i = 1, \ldots, r-2,$$

and notice that if $h_i \leqslant k$, then $b_{k-j} \notin \{a_{h_i+j}, \ldots, a_k\}$ for $0 \leqslant j \leqslant k - h_i$. Otherwise the subwalk $a_{h_i+j}, \ldots, a_k, z_1^i, \ldots, z_{h_i-1}^i, b_k, b_{k-1}, \ldots, b_{k-j}$ contains a cycle of length at most $k < g(G)$, which is a contradiction. This fact together with (2) and (3) allows to find a path from $A$ to $B$ in $P_k(G)$ by "shifting" all subwalks of length $k$ of $\mathscr{P}$ starting in $A$. Since the paths $Z_i$ are internally vertex-disjoint in $G$, it is evident that the corresponding induced paths $Z_i^*$ in $P_k(G)$ are internally vertex-disjoint in $P_k(G)$.

As regards $Z_{r-1}$ and $Z_r$ there are two cases to distinguish:

*Case* (a): Suppose that $z_1^r = a_{k-1}$ and $z_{h_r-1}^r = b_{k-1}$. Then $z_1^{r-1} \neq a_{k-1}$ and $z_{h_{r-1}-1}^{r-1} \neq b_{k-1}$ and therefore, the walk $\mathscr{P}_{r-1}$ induces a new internally vertex-disjoint path in $Z_{r-1}^*$ in $P_k(G)$. Thus it remains to find a path $Z_r^*$ in $P_k(G)$ joining $A$ with $B$ internally vertex-disjoint with $Z_i^*$ for each $i \in \{1, \ldots, r-1\}$.

Since $g(G) \geqslant k+1$, it follows that for each $j \in \{1, 2, \ldots, k\}$ there exist two paths $a_k = t_0, t_1, \ldots, t_j$, and $b_k = t_0^*, t_1^*, \ldots, t_j^*$ such that $t_j \notin \{a_j, \ldots, a_k\}$ and $t_j^* \notin \{b_j, \ldots, b_k\}$. In fact, we can take these paths in such a way that $t_1 \notin \{z_1^1, \ldots, z_1^{r-1}\}$ and $t_1^* \notin \{z_{h_1-1}^1, \ldots, z_{h_{r-1}-1}^{r-1}\}$, because $\delta(G) \geqslant r+1$. Let us consider the walk

$$w_0^r, w_1^r, \ldots, w_{n_r}^r = t_{k-1}, \ldots, t_0, z_1^r, \ldots, z_{h_r}^r, t_1^*, \ldots, t_{k-1}^*$$

of length $n_r = 2k - 2 + h_r$. We can find in $P_k(G)$ the following walk connecting $A$ with $B$:

$$\begin{aligned} Z_r^* : A = (a_0 a_1 \cdots a_k), (a_1 \cdots a_k t_1), \ldots, (a_{k-1} a_k t_1 \cdots t_{k-1}) &= (w_k^r \cdots w_0^r), \\ (w_{k+1}^r \cdots w_1^r), \ldots, (w_{n_r-k}^r \cdots w_{n_r}^r) &= (b_{k-1} b_k t_1^* \cdots t_{k-1}^*), \\ (b_{k-2} b_{k-1} b_k t_1^* \cdots t_{k-2}^*), \ldots, (b_0 \cdots b_k) &= B. \end{aligned}$$

Moreover, since $Z_r$ is internally vertex-disjoint with $Z_i$ in $G$ and besides $t_1 \neq z_1^i$ and $t_1^* \neq z_{h-1}^i$ for all $i = 1, \ldots, r-1$, we deduce that $Z_r^*$ is internally vertex-disjoint with $Z_i^*$, for all $i = 1, \ldots, r-1$.

*Case* (b): Suppose that $z_1^r = a_{k-1}$ and $z_{h_r-1}^r \neq b_{k-1}$, and that $z_1^{r-1} \neq a_{k-1}$ and $z_{h_{r-1}-1}^{r-1} = b_{k-1}$.

As $g(G) \geqslant k+1$ there exist two paths:

$$\begin{aligned} &a_k = t_0, t_1, \ldots, t_{k-1}, \quad \text{and} \quad b_k = t_0^*, t_1^*, \ldots, t_{k-1}^* \text{ such that} \\ &t_j \notin \{a_j, \ldots, a_k\} \quad \text{and} \quad t_j^* \notin \{b_j, \ldots, b_k\}, \quad \text{for each } j \in \{1, 2, \ldots, k-1\}. \end{aligned} \tag{4}$$

Moreover, we can take these paths in such a way that $t_1 \notin \{z_1^1, \ldots, z_1^{r-1}\}$ and $t_1^* \notin \{z_{h_1-1}^1, \ldots, z_{h_r-1}^r\}$ because $\delta(G) \geqslant r+1$. First we will construct the path $Z_r^*$.

Let us consider the walk formed by the union of the paths $\{t_{k-1}, \ldots, t_0\}$, $\{z_0^r, \ldots, z_{h_r}^r\}$ and $\{b_k, \ldots, b_0\}$, denoted by $w_0^r, w_1^r, \ldots, w_{n_r}^r$, thus $n_r = 2k - 1 + h_r$. We can construct in $P_k(G)$ the following walk connecting $A$ with $B$:

$$\begin{aligned} Z_r^* : A = (a_0 a_1 \cdots a_k), (a_1 \cdots a_k t_1), \ldots, (a_{k-1} a_k t_1 \cdots t_{k-1}) &= (w_k^r \cdots w_0^r), \\ (w_{k+1}^r \cdots w_1^r), \ldots, (w_{n_r-k}^r \cdots w_{n_r}^r) = (b_k \cdots b_0) &= B. \end{aligned}$$

Notice that $Z_r$ is internally vertex-disjoint in $G$ with the paths $Z_i$ and $t_1 \neq z_1^i$, for $i = 1, \ldots, r-1$. Thus $Z_r^*$ is internally vertex-disjoint in $P_k(G)$ with the paths $Z_i^*$. To construct a path $Z_{r-1}^*$ we proceed in an analogous way to find the path $Z_r^*$ changing $A$ with $B$. $\square$

## Acknowledgment

The authors wish to thank the referees for their helpful comments and suggestions which have allowed to improve the presentation of the paper.

## References


[1] R.E.L. Aldred, M.N. Ellingham, R.L. Hemminger, P. Jipsen, $P_3$-isomorphisms for graphs, J. Graph Theory 26/1 (1997) 35–51.
[2] C. Balbuena, D. Ferrero, Edge-connectivity and super edge-connectivity of $P_2$-path graphs, Discrete Math. 269 (2003) 13–20.
[3] C. Balbuena, P. García-Vázquez, Edge-connectivity in $P_k$-path graphs, Discrete Math. 286 (2004) 213–218.
[4] A. Belan, P. Jurica, Diameter in path graphs, Acta Math. Univ. Comenian LXVIII (2) (1999) 111–126.
[5] H.J. Broersma, C. Hoede, Path graphs, J. Graph Theory 13 (1989) 427–444.
[6] G. Chartrand, L. Lesniak, Graphs and Digraphs, third ed., Chapman & Hall, London, UK, 1996.
[7] G. Chartrand, J. Stewart, The connectivity of line-graphs, Math. Ann. 182 (1969) 170–174.
[8] M.A. Fiol, J.L.A. Yebra, J. Fabrega, Sequence graphs and interconnection networks, Ars Combin. 16-A (1983) 7–13.
[9] M. Knor, L. Niepel, Path, trail and walk graphs, Acta Math. Univ. Comenian. LXVIII (2) (1999) 253–256.
[10] M. Knor, L. Niepel, Connectivity of path graphs, Discuss. Math. Graph Theory 20 (2000) 181–195.
[11] M. Knor, L. Niepel, Diameter in iterated path graphs, Discrete Math. 233 (2001) 151–161.
[12] M. Knor, L. Niepel, Connectivity of iterated line graphs, Discrete Appl. Math. 125 (2003) 255–266.
[13] M. Knor, L. Niepel, M. Mallah, Connectivity of path graphs, Australas. J. Combin. 25 (2002) 174–184.
[14] H. Li, Y. Lin, On the characterization of path graphs, J. Graph Theory 17 (1993) 463–466.
[15] X. Li, The connectivity of path graphs, Combinatorics, Graph Theory, Algorithms and Applications, Beijing, 1993, pp. 187–192.
[16] X. Li, On the determination problem for $P_3$-transformation of graphs, Combinatorics and Graph Theory, vol. 1, Hefei, 1995, pp. 236–243.
[17] X. Li, On the determination problem for $P_3$-transformation of graphs, Ars Combin. 49 (1998) 296–302.
[18] X. Li, Z. Biao, Isomorphisms of $P_4$-graphs, Australas. J. Combin. 15 (1997) 135–143.
[19] J. Meng, Connectivity and super edge-connectivity of line graphs, Graph Theory Notes of New York, vol. XL, 2001, pp. 12–14.
[20] T. Zamfirescu, On the line-connectivity of line-graphs, Math. Ann. 187 (1970) 305–309.